## THE KING'S SCHOOL, CANTERBURY



## SCHOLARSHIP ENTRANCE EXAMINATION

## February 2012

## MATHEMATICS 1

## Time: 45 minutes (plus reading time)

Use the reading time wisely; gain an overview of the paper and start to think of how you will answer the questions.

Do as many questions as you can (clearly numbered) on the lined paper provided. Clearly name each sheet used. You are encouraged to attempt these questions in order.

The questions are not of equal length or mark allocation. Make sure you avoid spending too much time on any one question; don't get bogged down! Move on quickly if you get stuck. The paper is quite long; you are not necessarily expected to finish everything.

Some of the later questions are more difficult, but not necessarily longer. Some questions are designed to test your ability to work with unfamiliar ideas, or familiar ones with a twist. Don't give up!

You are expected to use a calculator where appropriate, but also you must show full and clear working, diagrams and arguments wherever you can. Marks will be awarded for method as well as answers. In fact, merely writing down an answer might score very few marks.

Complete questions are preferable to fragments. You can sometimes, however, manage to complete later parts of questions, even if you have failed to answer the earlier sections.

1 Harry and Abby buy tickets to see the London 2012 Olympics.

Harry buys three tickets for the 100 metres finals and two tickets to the opening ceremony and pays $£ 4187$.
Abby buys four tickets to the 100 metres finals (of the same type as Harry’s) and pays $£ 2900$.
How much does one of Harry's opening ceremony tickets cost?

2
Chidera sits an examination which has a possible raw mark total of $x$. She achieves a raw mark score of 81 which is equivalent to $x \%$.

What is the value of $x$ ?

3 In another school, the scholarship examinations for English and mathematics used to be the same length (i.e. time allocation). Nowadays the mathematics examination is $40 \%$ longer whereas the English examination has decreased in duration by one quarter of the increase in mathematics. The English examination is now a 45-minute paper.

How long is the mathematics examination now?

4 The Indian mathematician Srinivasa Ramanujan once approximated the circle constant $\pi$ with the calculation

$$
\sqrt[4]{9^{2}+\frac{19^{2}}{22}}
$$

(a) Work this out and show all the decimal places on your calculator.
(b) Using the $\pi$ button on your calculator, work out the error in the above approximation.
(c) What is this error as a percentage of the true value of $\pi$ ?

5 The equations

$$
\begin{aligned}
& 3 x-8=13 \\
& p x+17=3
\end{aligned}
$$

have the same solution. What is the value of $p$ ?

6 Suppose we define a new operation $\Delta$ as

$$
(x \Delta y)=\frac{x-y}{x+y}
$$

(a) Work out the value of $((1 \Delta 2) \Delta(3 \Delta 4))$.
(b) Work out in general the value of $((x \Delta x) \Delta y)$.

7


In an official factsheet giving information about the London 2012 Olympic Games, the following claim is made about the velodrome (cycling venue):
" 48,000 cubic metres of material were excavated to create the bowl for the velodrome - enough to fill 19 Olympic-sized swimming pools."

Let us check the arithmetic.

The dimensions of the main swimming pool in the Aquatics Centre are 50 m by 25 m with a depth of 3 m .

Assuming the pool is a cuboid, like this:


## NOT TO SCALE

(a) Work out how many of these pools could be filled by the volume of material excavated given above.
(b) Compare your answer to the original claim.
[Note for this question: 1 billion is 1000 million, and 1 trillion is 1000 billion]

UK Government debt has risen this January to a record $£ 1.004$ trillion from $£ 883$ billion a year ago.
(a) What percentage increase is this?

The current debt total of $£ 1.004$ trillion represents $64.2 \%$ of UK gross domestic product.
(b) Work out the UK gross domestic product. [Note that you do not need to know what gross domestic product means; it is just an amount of money.]
(c) How high would a single pile of $£ 20$ notes totalling $£ 1$ trillion be? Give your answer using sensible units [you may use the fact that one $£ 20$ note is 0.113 mm thick].
(d) Compare your answer to a length or distance you know in real life.

9 The table below shows the percentage of UK school pupils gaining a grade A*-C when sitting any single GCSE examination.

Showing your reasoning and calculations clearly, work out an estimate of the year in which $100 \%$ of students will gain $\mathrm{A}^{*}-\mathrm{C}$ in any single GCSE examination.
[You may use graph paper but it is not necessary to do this to achieve full marks.]

| Year | \% A*-C |
| :--- | :---: |
| 2010 | 58.4 |
| 2009 | 57.2 |
| 2008 | 56.3 |
| 2007 | 55.2 |
| 2006 | 54.3 |
| 2005 | 53.4 |
| 2004 | 51.7 |
| 2003 | 50.2 |
| 2002 | 51.3 |
| 2001 | 50.1 |
| 2000 | 49.2 |
| 1999 | 48.1 |

## END OF PAPER

